

# WG4 | Mathematics in Games

## In-Person Meeting

Day 0 | Let's Play

This is the virtual program for the first In-Person Meeting of the Workgroup on Mathematics in Games. The meeting is part of the [COST action CA22145 Computational Techniques for Tabletop Games Heritage](#).

It is held on Friday, March 15, 2024, in Aveiro, Portugal in person at Fabrica and online over Zoom.

Address: R. dos Santos Mártires 1A, 3810-164 Aveiro, Portugal.

Access link for Zoom here: <https://univ-nantes-fr.zoom.us/j/85939596222>

The meeting follows the (Portuguese) [National Championship of Mathematical Boardgames](#), the day prior.

We kindly thank the hosting venue, [Fábrica](#) of the Portuguese Sci-Com network [Ciência Viva](#), and the [University of Aveiro](#), for receiving us.

# Schedule

Please note that all times are Lisbon time, WET.

09:00            Opening

## Session 1

09:10    *What do we mean by "Games & Math"*

Jorge Nuno Silva

09:30    *A quick journey into Combinatorial Game Theory*

Alda Carvalho

09:50    *What's Math Got To Do With It? Board Games In The Deep Past*

Walter Crist

10:10    *What is Automated Game Design and Why Does It Need Mathematicians*

Younès Rabii

10:30 - 11:00    Coffee-break

## Session 2

11:00    *Digital Topology and Board Games*

Emin Durmishi

11:20    *Exploring the Integration of Education, Mathematics, and Tabletop Games*

Elif Öztürk

11:40    *Drama in the game of Goose*

João Pedro Neto

12:00    *Ludii's Mathematical Metrics and Playability*

Eric Piette

12:30 - 14:00    Lunch

## Session 3

14:00    *Ludus Regularis*

Carla Cardoso

14:20    *A principled approach to intransitivity and cyclicity in paired comparisons data*

Ori Davidov

14:40    *Positional Games*

Miloš Stojakovic

15:00 - 15:20    Break

## Session 4

15:20    *Pursuit games on graphs*

Jessica Enright

15:40    *Finding portfolios of strategies in the large imperfect information games*

Karolina Drabent

## Abstracts

### What do we mean by "Games & Math"

Jorge Nuno Silva, Associação Ludus, Portugal

We'll survey the many relevant relations between games and mathematics. Some paradigmatic examples will be presented to illustrate the scientific areas that are fit to play a part in the research of games and their cultural heritage.

### A Quick Journey Into Combinatorial Game Theory

Alda Carvalho, Universidade Aberta, Portugal

Abstract: In the early years of the 20th century, C. Bouton presented a famous analysis of the classic game of NIM. Later, in the 1930s, independently, R. Sprague and P. Grundy outlined the method for analyzing impartial games, where allowable moves depend only on the position and not on which of the two players is making the move. Even later, in the 1970s, E. Berlekamp, J. Conway, and R. Guy delved into partizan games, with positions where one of the players may have options that the opponent does not. These three developments contributed to the establishment of a new mathematical subject: Combinatorial Game Theory. The aim of this talk is to provide a survey of the development of this mathematical field, focusing solely on the normal-play winning convention, which establishes that the player who runs out of moves loses. The speaker will complement the talk with some well-known examples and personal insights.

### What's Math Got To Do With It? Board Games In The Deep Past

Walter Crist, Universiteit Leiden, Netherlands

Researchers have, for a long time, attempted to connect the development of board games with the development of milestones in mathematical thinking or representation in ancient societies. Indeed, some include the very invention of board games as one of these mathematical milestones that could not be possible without the achievement of some kind of formalized mathematics. Others suggest that increased complexity of board games as observed through time can be connected to changes in mathematical thinking, using evidence such as the development of different kinds of dice and movement in two rather than one dimension as evidence of this. This presentation presents very preliminary thoughts on the challenges of examining board games from this perspective, with the purpose of stimulating conversation about how we can learn more about the deployment of mathematical ideas in ancient board games.

### What is Automated Game Design and Why Does It Need Mathematicians

Younès Rabii, Queen Mary University of London, United Kingdom

In this talk, Younès will briefly introduce the field of automated game design and the concept of an automated game designer.

They will present some of the work that was produced in the past ten years, the goals of the field and the challenges it is currently facing.

Younès will also focus on specific problems that could greatly benefit from having mathematicians looking at them – with the hopes of sparking collaborations!

## Digital Topology and Board Games

Emin Durmishi, University of Tetovo, North Macedonia

Digital topology serves as a fundamental tool in analyzing topological properties within image processing. In this presentation, we explore the possible application of digital topology within the realm of board games. Through this application, we offer insights into generating new variants of some specific territorial board games.

Beginning with an elucidation of key concepts in digital topology, including 4-adjacency and 8-adjacency, we delve into resolving the Jordan curve paradox in digital topology, elucidate the notions of a set's boundary and interior, and explore the Khalimsky topology alongside 6-adjacency.

We propose practical implementations of digital topology to reimagine classic table games such as Go and Conway's Game of Life.

## Exploring the Integration of Education, Mathematics, and Tabletop Games

Elif Öztürk, Orta Doğu Teknik Üniversitesi, Turkey

In my presentation, I will explain the intersection of my research background in adaptive expertise in education with tabletop games in mathematics, enriching our exploration within the WG4. To briefly define adaptive expertise, it refers to the ability to apply knowledge and skills flexibly, effectively, and creatively across diverse contexts. Individuals with adaptive expertise can navigate novel situations, solve complex problems, and transfer knowledge seamlessly. This concept underscores not just possessing knowledge but also the ability to utilize it in dynamic and changing environments.

By incorporating adaptive expertise into my discussion, I will highlight the symbiotic connection between my students' mathematical knowledge, their adaptive expertise skills and the complexity of their game creations that I gave as final project assignments in my courses. Indeed, students with adaptive expertise capabilities and a strong math foundation may implicitly indicate a deeper understanding, effortlessly into their tabletop game designs. Conversely, those less familiar with mathematical concepts may find inspiration in popular existing games. In fact, adaptive expertise capabilities of students will make them be able to transfer their mathematical knowledge to their game designs to reinforce mathematical concepts such as probability, geometry, and arithmetic in their tabletop games.

Overall, the presentation will briefly overview my research, explain the ways of increasing adaptive expertise skills of students emphasizing how these studies can deepen our comprehension of games and mathematics.

## Drama in the game of Goose

João Pedro Neto, University of Lisbon, Portugal

In games, drama is linked to the possibility of recovering from a seemingly weaker position. Drama is a feature that arguably good board games should have, since it is relevant in the perception of the play experience as pleasant. Despite its intrinsic qualitative nature, we suggest the adaptation of the concept of drama to games of pure chance and propose a set of objective criteria to measure it.

## Ludii's Mathematical Metrics and Playability

Eric Piette, Université catholique de Louvain, Belgium

This presentation will delve into the Ludii software, a versatile General Game System encompassing over 1300 games. Ludii not only plays and evaluates games but also aids in game design across various genres such as board games, dice games, and mathematical games. Its diverse game library continues to expand through ongoing research projects, including PhD theses on Card games and numerous undergraduate projects on 3D games and puzzles at my institution. Additionally, collaborations with other institutions are producing projects such as the automatic generation of original and engaging new board games. In Ludii, to assess playability and analyze these games, we employ a range of game metrics established during the previous ERC-funded Digital Ludeme Project. However, we have yet to thoroughly explore the mathematical descriptions of these games. This presentation aims to spark discussion on the topic: "How can we mathematically define and compute the playability of games?"

## Ludus Regularis

Carla Cardoso, Associação Ludus, Portugal

Ludus Regularis Seu Clericalis, the Clergy Game, was invented in the 10th century by Bishop Wibold of Cambrai (France), with the goal of discouraging the involvement of clerics in gambling and make them experience the pleasures of the practice of a dice game with a virtuous theme.

The game's rules were kept by the historian and bishop of Noyon and Tournai in the 11th century, Balderic of Théroutan, edited in 1615 in a book by Colvener: *Chronicon Cameracense et Atrebatense*, and republished in 1834 by André Le Glay, in Paris: *Chronique d'Arras et de Cambrai*.

In the work that has now been carried out, the game has been probabilistically analyzed and some of the myths about the symbolism surrounding it have been exposed. For the time being, the research seems to be over, but perhaps there will be new lines of study, which have been kept since the 17th century in the north of France. The present study on Ludus Regularis is part of my present PhD work in History of Science, at the University of Lisbon.

## A principled approach to intransitivity and cyclicity in paired comparisons data

Ori Davidov, University of Haifa, Israel

A principled approach dealing with intransitivity and cyclicity for cardinal paired comparison data is developed. Although the focus is on graphical linear models for cardinal paired comparison data the approach is readily extended to binary and other types of paired comparison data. Goodness of fit tests for detecting local and global lack of fit are proposed. The structure of the parameter space, which accommodates intransitivity and cyclicity, is rigorously studied. A new and novel model selection method, which builds on the properties of the aforementioned parameter space is proposed. The large sample properties of the estimators as well as guarantees on the selected model are studied.

## Positional Games

Miloš Stojakovic, University of Novi Sad Faculty of Sciences, Serbia

The Theory of Positional Games is a fairly independent branch of Combinatorial Game Theory, nested between Theoretical Computer Science and Mathematics, with numerous applications in both. It deals with a class of two-player perfect-information games, ranging from popular games such as Tic-Tac-Toe, Hex and Sim to some purely abstract games played on graphs and networks. Our hope is to present a short overview of the field.

## Pursuit games on graphs

Jessica Enright, University of Glasgow, United Kingdom

Pursuit games on graphs include a variety of cop-and-robber style games where an evader tries to escape a chaser. Researchers have investigated a wide variety of variants including different speeds or levels of knowledge of the agents, or graph characteristics. I'll introduce some classic variants, and mention more recent work on a version with designated special nodes that can help the evader escape. There will be an algorithmic slant here, with discussion of the computational complexity of resolving the outcomes of these games.

## Finding portfolios of strategies in the large imperfect information games

Karolina Drabent, Ceske Vysoke Ucení Technické V Praze, Czech Republic

In the domain of large imperfect information games, such as poker or Stratego, computing the Nash equilibrium poses significant computational challenges. To address this, popular solutions often employ methods to abstract the vast game space into more manageable forms. This abstraction extends to strategies, forming what is known as a portfolio of strategies. Our research delves into the exploration of portfolio selection within the realm of normal form games. Specifically, we focus on identifying portfolios that are both effective for exploiting opponents and robust in varied game scenarios. We seek to advance the theoretical understanding and practical application of strategic decision-making in complex, multi-agent environments.